Factoring Very Very Large Numbers

A few problems to take with you for the winter break.

1. The sum of the digits of 100! = 1 x 2 x 3 x ... x 99 x 100 has been written as a (very very large) whole number. The sum of its digits is calculated, which results in another whole number. The sum of those digits are then calculated. This process can be repeated until the result is a one-digit number. What is that number? *Hint: We almost solved this. Try it for 11!, then 12!, then 13!.*

2. Cross out one of the factors of the form n! in the product (1!)(2!)(3!)...(99!)(100!) so that the remaining product is a square number. You might want to try to solve the same problem for "smaller" products first, like (1!)(2!)(3!)(4!), or (1!)(2!)(3!)(4!)(5!)(6!), or (1!)(2!)(3!)(4!)(5!)(6!)(7!)(8!).

3. Which factorials can be written as products of smaller factorials? For example, we can write 10! = (7!)(5!)(3!).