Title: Fractal Frenzy

Warm-Up

1. We are going to create a sequence of images.

   Here is the first "term" in the sequence:

   Here is the second "term" in the sequence:

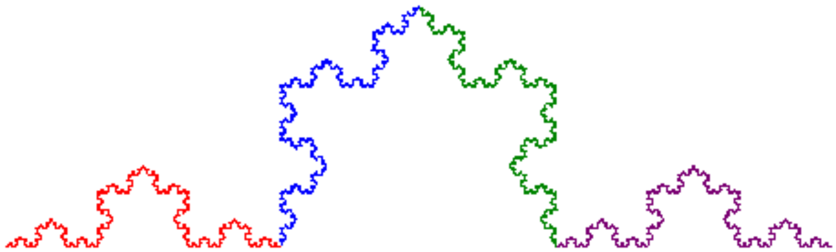
   Do you see a pattern? Try to draw the 3rd and 4th "terms" for the sequence.

   Third:

   Fourth:

2. **General Discussion of Fractals**

What we started to build above is called the **Koch curve**, named after the famous Swedish mathematician *Niels Fabian Helge von Koch***.** Why is it called a curve?
Think about what happens after many iterations.



The Koch curve is an example of a **fractal**. Fractals are based on the idea of self-similarity; any small part within the larger image has a similar pattern to the larger image.

Here are a few other examples:

For each fractal that we talk about today, we will build it in a sequence of steps. Let's call the first step the **base** and the second step the **motif**. Once we know the **base** and **motif**, we can look at some of the following steps. (We won't look at all of the steps because there are an *infinite* number of steps.)

For example, the base and the motif for the Koch curve are:

Step 1 (Base)

Step 2 (Motif)

3. The Cantor (Middle Third) Set Now we are going to look at another famous fractal named after the German mathematician Georg Cantor.
4.
   - Start with a long line segment.

   - Draw several more copies (at least 4) of this line segment directly below it (going down the board).
   - Label the first segment Step 1, the second Step 2, the third Step 3, etc.
   - Now divide the Step 2 segment into three equal parts.

   - Erase the middle third. You will have one line segment on the left and a different one on the right with a space in the middle that is the exact same length.

   Repeat this for the rest of your segments, except for Step 1.
   - Now move on to Step 3. Divide EACH segment into three equal parts, and then erase the middle third of each.
   - Continue this pattern to complete the next steps.

   - Some questions to answer:

   **What is the "base" and what is the "motif" for this fractal?

   **What will happen if you keep going?

   **Now complete as much of this table as possible.

Cantor's Middle Third	**Total Number of Line Segments**
Step 1	
Step 2	
Step 3	
Step 4	
Step 5	
Step 6	

   **Do you see a pattern in the table above? If so, write a formula to describe it.

4. **<u>The Sierpinski Triangle</u>** The fractal we are looking at here is called the **Sierpinski triangle**, named after Polish mathematician Wacław Sierpinski. Here are the base and motif for this fractal:

Step 1 (Base)

Step 2 (Motif)

- Use a straight edge and triangular grid paper to draw a larger version of Step 1. (I suggest making it an equilateral triangle with each side=8 units. Unit=length of one side of each triangle in the grid.)
- *Lightly shade* in the entire large triangle using a pencil.
- Make 3 more copies of this using the front a back of your grid paper.
- Label your shaded triangles with step numbers: Step 1, Step 2, Step 3, Step 4.
- On the triangle labeled Step 2, use an eraser to help you create the motif image shown above. Look at the image carefully and be exact. Ask an adult to check your image and then repeat the same process on the triangles labeled Step 3 and Step 4.
- Use your eraser to make Step 3 triangles on your remaining two copies.
- Finally, we are at Step 4. Use your eraser to show what the Step 4 triangle looks like.

  We could keep going and going, but I think we get the idea! ☺️

- Use your images of the Sierpinski Triangle to fill in the table below:

The Sierpinski Triangle	**White Triangles**	**Shaded Triangles**
Step 1		
Step 2		
Step 3		
Step 4		
Step 5		

Some More Questions:

We are only counting the perimeter and area of the ***shaded*** triangles. Use your pictures of the first four steps to help you.

****Perimeter**

- Fill in the "Perimeter (Triangles)" column for Steps 1-4 below. This means you should find the perimeter using the length of one triangle on the grid as your unit. For example, if you look at Step 1 each side should be the length of 8 triangles on the grid.
- Now we will represent the length of each side of the large triangle as ***s***. That is, ***s***, is equal to the length of 8 triangles on the grid paper. Find the perimeter of the fractal in each stage in terms of ***s***. Write your answers in the "Perimeter (s)" column.
- See if you notice a pattern. Use that pattern to predict the perimeter of the Step 5 Sierpinski Triangle. Show work and fill in the values in the table.

The Sierpinski Triangle	Perimeter (Triangles)	Perimeter (s)
Step 1		
Step 2		
Step 3		
Step 4		
Step 5		

****Area:** Now we will do the same for area.

- Use the area of one triangle on the grid as your unit. This is tricky! Look for patterns and shortcuts to help you. Fill in the Area (Triangles) column.
- Represent the area of the large triangle as A. This means that A is equal to the area of 169 triangles on the grid.
- Once you have steps 1-4 done make a prediction for Step 5.

The Sierpinski Triangle	Area (Triangles)	Area (A)
Step 1		
Step 2		
Step 3		
Step 4		
Step 5		

5. **Koch Snowflake**