Games with Ones and Zeros

PROBLEM 1. Five students sat in a row of five chairs, facing sideways. We called them, from right to left, persons 1 through 5.

They could stand and sit according to the following rules:

a. Person number 1 can stand or sit at any time.
b. Person number 2 can stand or sit only if the first person is standing.
c. Person number 3 can stand or sit only if the second person is standing and the first person is sitting.
d. Persons 4 and 5 can change position (sitting or standing) if and only if the person before him/her is standing, and everyone with a lower number is sitting. (Note that this is a more general version of rules a, b, and c.)

The object of the game is to get the fifth person standing!

PROBLEM 2. We took the 8 subsets of {a,b,c} and wanted to arrange them in a sequence so that each subset differs from the previous one in having exactly one new element added or exactly one old element deleted.

Here is an example of one way to list these in such a sequence:

{b} , { } , {a} , {a, b} , {a, b, c} , {a, c} , {c} , {b, c}

a. In how many ways can this be done?
b. Does this change if we insist that the list be a 'loop' - that is, that the last element can lead 'legally' back to the first element?
c. We asked the same questions for two elements {a,b}, and for four elements {a,b,c,d}.
d. We noticed that {a,b,c} was related to the symmetries of the cube, {a,b} the symmetries of the square, and {a,b,c,d} the symmetries of the hypercube.

HOMEWORK PROBLEM. If we continued to play the game in Problem 1 with only one person changing from standing to sitting or vice verse in each stage, and listed the people standing and sitting at each stage in a column, with a 1 for “standing” and a 0 for “sitting”, what patterns if any do we see of 0’s and 1’s in each column? Are there any patterns in the rows?