Title: *More Than Just the Sum of Its Parts*

Source: Vandervelde, Sam. "Circling the Square." *Circle in a Box*, Mathematical Sciences Research Institute, 2009.

<u>Guiding Problem and Activities</u>

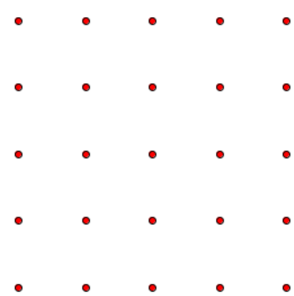
1. Find the value of the following sum without using a calculator.

$$1 + 2 + 3 + \ldots + 2019 + 2020 + 2019 + \ldots + 3 + 2 + 1$$

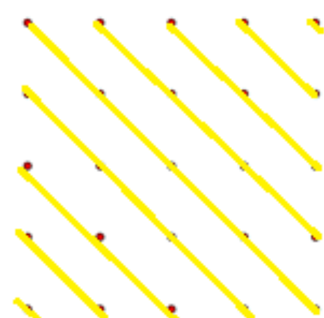
- Did anyone reach a final answer? If so, <u>why</u> is this the answer?

- Let's look at a simpler problem. For example:
  - $1 + 2 + 3 + 2 + 1$
  - $1 + 2 + 3 + 4 + 3 + 2 + 1$
  - $1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1$
  - What pattern do you notice? How can we apply this to the original problem?

- How do we represent perfect squares geometrically? For example, the number 25 can be represented as an array of dots:



  - How can we represent the quantities 1, 2, 3, 4, 5, 4, 3, 2, 1 on this array? How does this relate to the value of the sum 1+2+3+4+5+4+3+2+1?



- Based on this evidence, can you form a conjecture about the value of the sum

$$1 + 2 + 3 + \ldots + (n-1) + n + (n-1) + \ldots + 3 + 2 + 1$$

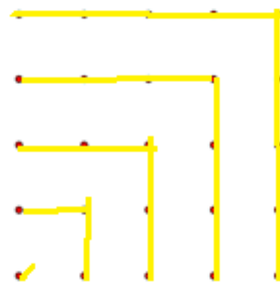
if n is any positive whole number?

Further Questions/Problems

2. List as many perfect squares as you are able to. What facts do you know about perfect squares? Can you give an example to support each fact?

3. What is the sum of the first few odd numbers? More specifically, what is the sum of the first four (1, 3, 5, 7)? Does this sum relate to perfect squares? Or, is there a different way that you see it?

   Here is an interesting example for 1+3+5+7:



4. Find the values of 15^2, 25^2, 35^2, 45^2, etc. .
   a. Do you notice a pattern with the last two digits of the answers? In the first two digits?
   b. Can you use a picture to explain the pattern that you see? (Here's an example of where you could start with 25.)