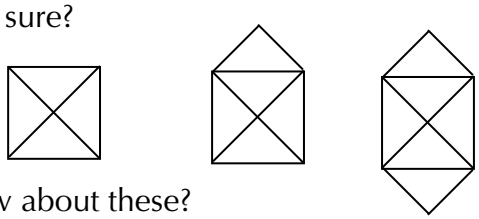
We discussed whether you could draw a particular shape without lifting your pencil or retracing lines. These are called Euler circuits or Euler paths (dependent on whether you do or do not start and end at the same point, respectively). Leonard Euler was a 18th century Swiss mathematician. Some of the insights Euler had were due to solving the Seven Bridges of Königsberg problem. For more information on these topics, visit:

Euler Paths and Circuits:
http://discretetext.oscarlevin.com/dmoi/sec_paths.html
https://en.wikipedia.org/wiki/Eulerian_path
Leonard Euler:
http://www-groups.dcs.st-and.ac.uk/history/Biographies/Euler.html
https://en.wikipedia.org/wiki/Leonhard_Euler
Seven Bridges of Königsberg:
https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg

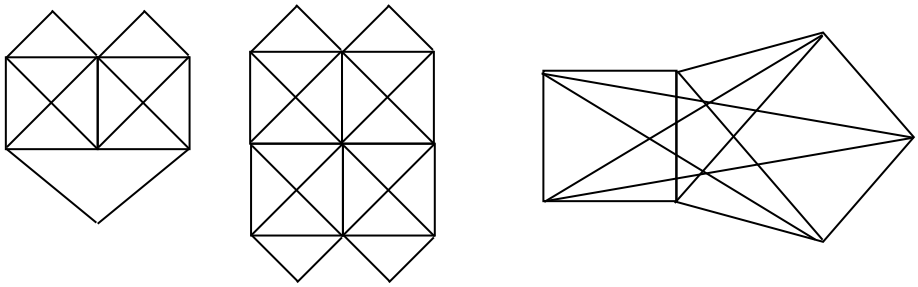
Additional Questions:
Can you make a graph that has only one vertex of odd degree?
Can you start at ANY vertex if all vertices have even degree?
Is there an algorithm (a set of instructions you could give to a computer) to figure out how to find the Euler path or circuit given the appropriate graph?

Exercises

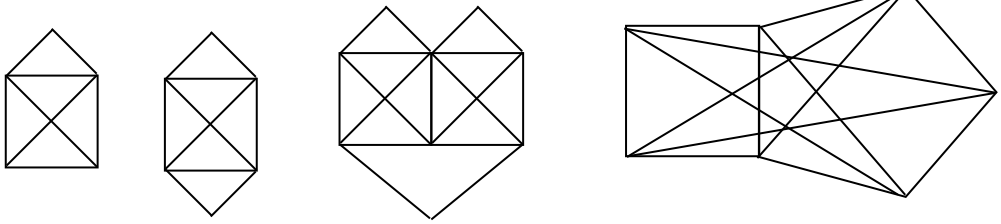
1. Which of these can you draw without retracing any lines or lifting your pencil? Are you sure?



2. How about these?

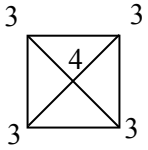


3. Goal of what we're doing today – know within 1 minute if each of these can be drawn without lifting your pencil! We figured out (or I'm telling you) the following ones can be done.



Here's what to do with your group: All of these things are called "graphs." A place where the lines cross is called vertex (or vertices for plural). Start at one vertex and mark it with a "S." Then when you're done, mark the last vertex with an "E." If you start and end at the same vertex, you can mark it "SE" or "ES." If you can't figure out some of them, that's OK.

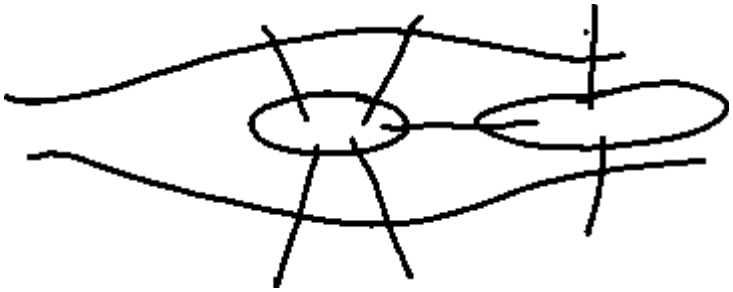
4. The lines in between vertices are called "edges." Let's count how many edges touch each vertex.



Now do it for all the other shapes we've talked about.

5. Which graphs can be drawn without retracing or lifting a pencil?

6. Seven Bridges of Königsberg.



The goal is to cross each bridge exactly once. Can it be done? If so, how?

7. What if we converted this to a graph? What would the graph look like?