October 3, 2017
Math Circles
American University, Washington, DC
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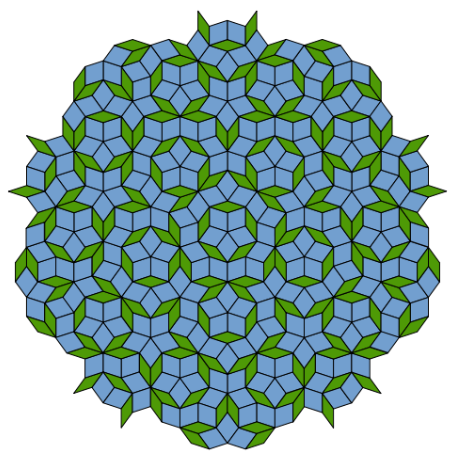
For parents, participants, or other interested people:

Tonight, I decided to try two smaller activities for the kids rather than one longer one, to hold their focus more consistently.

TOPIC ONE:

Penrose Tiles: https://en.wikipedia.org/wiki/Penrose_tiling This article on Wikipedia is a great introduction to Penrose Tiles. Rodger Penrose took this more-or-less esoteric topic and brought it down to the level of a toy. The underlying mathematical question is to construct a tiling that covers the plane, but never repeats. Can this be done? Well, if certain rules are followed, he showed that we can promise it can be done even though we will never be able to construct it.

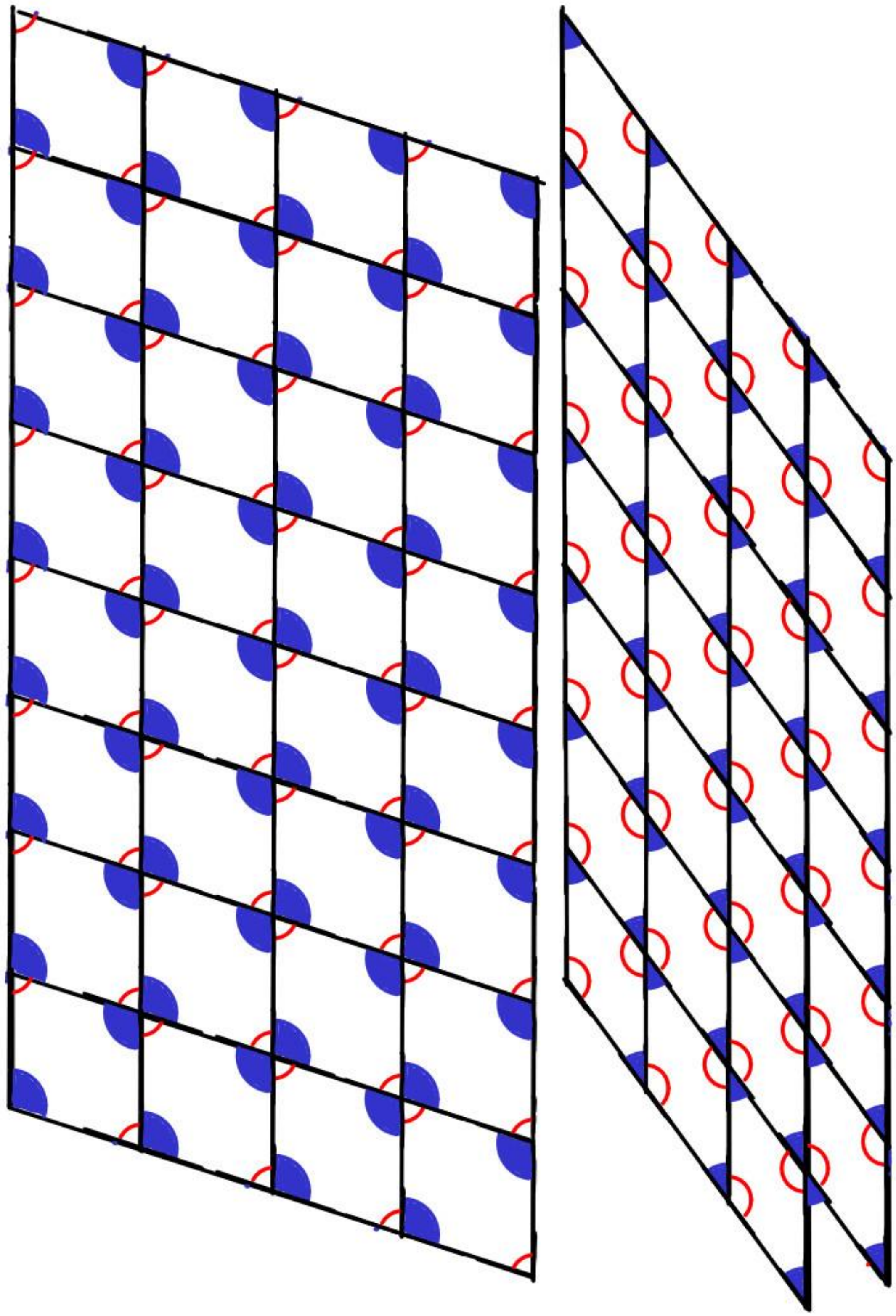
When most people (mathematicians or not) are confronted with Penrose tiles, the first "fail" that often happens is that the tiles are not marked with their matching rules. In this case, people completely fail to grasp the idea, and they simply create some easily translated sequence of pieces which tiles the plane, and they think they understand. The second "fail", in my estimation, is that this pattern emerges:



Which, to me, is a "fail", because although technicallly – it cannot be translated in order to be repeated, it has a different sort of symmetry, and there is no mystery or true irregularity in its construction. So even though it's beautiful, I think it totally misses the entire point of having the discussion in the first place.

So, to get the kids thinking in the way I wanted them to be thinking about this, I started with those points at which the tiles meet, called "vertices". I had them construct all 9 legal vertex types. Then, in order to truly capture the essence of Penrose Tilings, I had them attempt to merge all 9 types of vertices into one single construction.

I feel that went well. I feel they got the point. (On the next page, I include the template I gave them.)



TOPIC TWO:

A variation on Nim. Again, I urge those who are interested to read more on Wikipedia. https://en.wikipedia.org/wiki/Nim

In this variation, only one pile of goods was used. I tossed dice to determine the number of markers/candies to be wagered, and another die to determine who started the game. I've done this game with freshmen in my "Great Ideas" course, telling them they can pick anywhere from x to y number of candies, but to make it easier for the younger kids, I just set that to be "1 to 4" and did not vary it throughout the evening. The kids alternate "stealing candies" and the last one to take a candy loses.

The solution to this game is one that many of the kids figured out (as I expected and hoped), which is to first notice that if your opponent leaves you with only one candy, you must lose.

Quickly, they figure out that just like 1 is a losing situation, so is 5, because your opponent will simply take 4 and leave you with 1. Consequently, if you see 2, 3, 4, or 5, you take enough to leave one for your opponent. So, you want any number other than 1, 6… or what else?!

After a few more rounds, they figure out the pattern.

see	take		see	take		see	take
1	(lose)		7	1		13	2
2	1		8	2		14	3
3	2		9	3		15	4
4	3		10	4		16	(lose)
5	4		11	(lose)		17	1
6	(lose)		12	1		18	2

This is a real-world example (ok, it's really contrived, but to them it is real) of modular arithmetic. All the numbers equal to *1 mod 5* are losing situations, while the rest are winning. (The expression "1 mod 5" means you have a remainder of one when you divide that value by 5.) Once you get yourself into a winning situation, in order to maintain it, you need only to mirror your opponent's move by keeping your modular value the same. If your opponent takes 1, you take 4. If your opponent takes 2, you take 3. By using this technique, you continually subtract off 5's to keep the same remainder when dividing by 5.

When there is more time to explore this game, you can make a new variation where you can take only 1 or 2, for example (or any range of values is fine). If you add the top and bottom number together, that is the modular system you will be bound to. For example, if you allow someone to take between 2 and 8 candies, the modular system will be 10 and you will try not to be given anything equal to *1 mod 10*.

So, for parents, this is one way you can extend the ideas learned in Circles to play the game at home and keep the learning going!