Notes from January 23, 2018

DC Math Circle

American University

Department of Mathematics and Statistics

I'm thinking of a famous math sequence.

It starts with 1, 2, 3, 5.

Do you know what the sequence is?

That's a trick question!

There are tons of famous math sequences that start that way.

There is a website called oeis.org that is all about famous math sequences.

THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

founded in 1964 by N. J. A. Sloane

1,2,3,5 Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search: **seq:1,2,3,5**

Displaying 1-10 of 3539 results found. page 1 2 3 4 5 6 7 8 9 10 ... 354

Sort: relevance | references | number | modified | created Format: long | short | data

A000045 Fibonacci numbers: F(n) = F(n-1) + F(n-2) with F(0) = 0 and F(1) = 1. +20
(Formerly M0692 N0256) 4193

0, 1, **1**, **2**, **3**, **5**, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,4

COMMENTS Also sometimes called Lamé's sequence.

It says that the most famous sequence starting with 1, 2, 3, 5 is the Fibonacci numbers.

But it also lists 3538 other sequences that start with 1, 2, 3, 5!

Here's another one.
It is called the partition sequence...

1
$\underbrace{1}$
1 partition of 1

2
$\underbrace{1+1, 2}$
2 partitions of 2

3
$\underbrace{1+1+1, 1+2, 3}$
3 partitions of 3

4
$\underbrace{1+1+1+1, 1+1+2, 2+2, 1+3, 4}$
5 partitions of 4

Problem Figure out how many partitions there are for 5, 6, 7, 8, 9, 10.

Fill in the table...

number	number of partitions
1	1
2	2
3	3
4	5
5	
6	
7	
8	
9	
10	

You can actually look up the answers on oeis.org ...

A000041 a(n) = number of partitions of n (the partition numbers). +20
(Formerly M0663 N0244) 2178

1, **1**, **2**, **3**, **5**, 7, 11, 15, 22, 30, 42, 56, 77, 101, 135, 176, 231, 297, 385, 490, 627, 792, 1002, 1255, 1575, 1958, 2436, 3010, 3718, 4565, 5604, 6842, 8349, 10143, 12310, 14883, 17977, 21637, 26015, 31185, 37338, 44583, 53174, 63261, 75175, 89134, 105558, 124754, 147273, 173525 (list; graph; refs; listen; history; text; internal format)

OFFSET 0,3

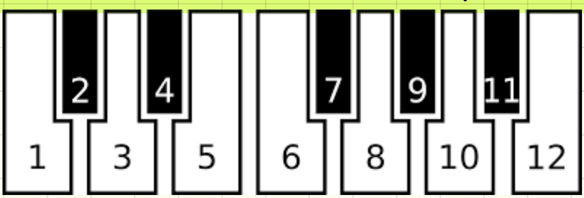
COMMENTS Also number of nonnegative solutions to b + 2c + 3d + 4e + ... = n and the number of nonnegative solutions to 2c + 3d + 4e + ... <= n. - Henry Bottomley, Apr 17 2001
a(n) is also the number of conjugacy classes in the symmetric group S_n (and the number of irreducible representations of S_n).

Most of the stuff of oeis.org is really, really complicated!!

So it is cool when you can figure out what some of it means.

Do you see where it says "listen"?

oeis will convert numbers in a sequence into musical notes on a piano keyboard.



Then you take the notes and make a song with them with a program like Garage Band.

That is how I made the song in the cartoon today!

Here is a funny sequence...

1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 11, 21, 31, ...

You can look that up on oeis if you can't figure it out!

Now here is another very famous sequence...

1, 3, 6, 10, 15, 21, 28, ...

There are lots of ways to describe the pattern in this sequence.

Can you think of some?

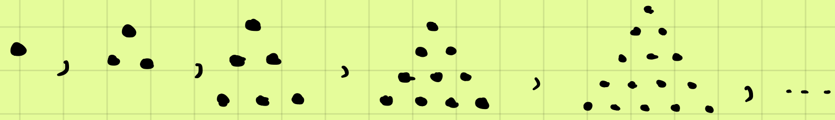
Here are some of my answers ...

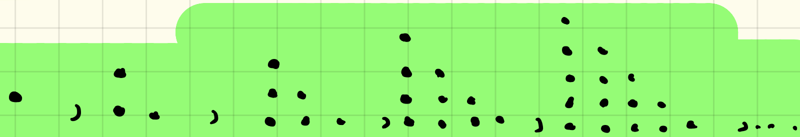
$1 + 2 + 3 + \dots + n$

$1,\ 1+2,\ 1+2+3,\ 1+2+3+4,\ 1+2+3+4+5, \dots$

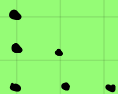
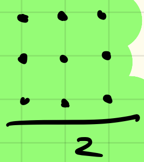
$\frac{1\times 2}{2},\ \frac{2\times 3}{2},\ \frac{3\times 4}{2},\ \frac{4\times 5}{2},\ \frac{5\times 6}{2}, \dots$

$\frac{n(n+1)}{2}$



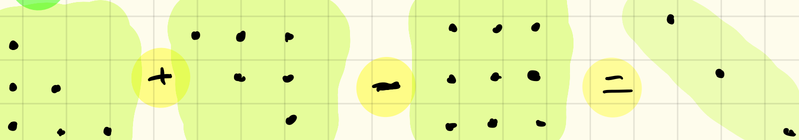


The last sequence looks like it has half-squares.

In other words,  look like .

But that is not quite right.
The triangle has 6 dots and the square has 9 dots.

What happens in general if you take 2 times $1 + 2 + 3 + \dots + n$ and subtract $n \times n$?



Now here is another sequence that is similar to the previous one...

$1, 5, 14, 30, 55, \ldots$

Do you see any pattern?

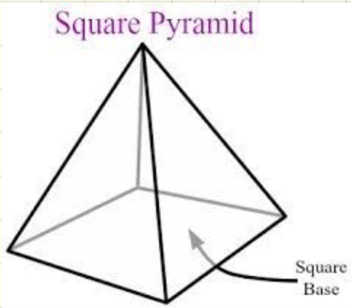
I'll tell you one.

$$\underset{\underset{1}{\parallel}}{1^2},\ \underset{\underset{5}{\parallel}}{\underbrace{1^2+2^2}},\ \underset{\underset{14}{\parallel}}{\underbrace{1^2+2^2+3^2}},\ \underset{\underset{30}{\parallel}}{\underbrace{1^2+2^2+3^2+4^2}},\ \ldots$$

On oeis these are called "square pyramidal numbers."

Can you guess why?

Hint: Here is what a square pyramid looks like



Remember how

$$1+2+3+4+5 = \frac{5\times 6}{2} \quad \text{and} \quad 1+2+\cdots+n = \frac{n\times(n+1)}{2}\ ?$$

Well there is something like that for $1^2+2^2+\cdots+n^2$.

Instead of $\frac{n\times(n+1)}{2}$, it is $\frac{n\times(n+1)\times(\text{mystery number})}{6}$.

What is the mystery number?

If you like this stuff, you can also think about $1^3, 1^3+2^3, 1^3+2^3+3^3, \ldots$ and sequences for higher powers.

Here is another cool problem.
Make a table

n	$1+2+\cdots+n$	$1^3+2^3+\cdots+n^3$
1	1	1
2	3	9
3	6	36
4	10	100
5	15	225

What is the relation between the numbers in the 2nd and 3rd columns?
Does the pattern continue?

(For more info, read about Nichomachus's Theorem.)

These problems are related to the following triangle of numbers

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & \frac{1}{2} & & \frac{1}{2} & & \\
 & \frac{1}{3} & & \frac{1}{2} & & \frac{1}{6} & \\
\frac{1}{4} & & \frac{1}{2} & & \frac{1}{4} & & 0 \\
\vdots & & \vdots & & \vdots & & \vdots
\end{array}
$$

$$
\begin{array}{c}
n \\
\frac{1}{2}n^2+\frac{1}{2}n \\
\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n
\end{array}
$$

Do you see any patterns?